On economic growth and minimum wages

Luciano Fanti and Luca Gori

Department of Economics, University of Pisa, Department of Economics, University of Pisa

12. October 2010

On economic growth and minimum wages

Luciano Fanti[*] and Luca Gori[**]

Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy

Abstract We offer an analysis of the existence of a positive relationship between minimum wages and economic growth in a simple one-sector overlapping generations economy where the usual Romer-typed knowledge spill-over mechanism in production represents the engine of endogenous growth, in the case of both homogeneous and heterogeneous (i.e., skilled and unskilled) labour. Assuming also the existence of unemployment benefits financed with consumption taxes not conditioned on age at a balanced budget, it is shown that minimum wages may stimulate economic growth and welfare despite the unemployment occurrence. Moreover, a growth-maximising minimum wage can exist. A straightforward message, therefore, is that a combination of minimum wage and unemployment benefit policies can appropriately be used to promote balanced growth and welfare.

Keywords Endogenous growth; Minimum wage; Unemployment; OLG model

JEL Classification H24; J60; O41

We are indebted to Prof. Eric Maskin, Prof. Davide Fiaschi, seminar participants at the 19th Jerusalem School in Economic Theory "The Economics of Kenneth J. Arrow", held on June 27–July 8, 2008 at the Institute for Advanced Studies, Hebrew University of Jerusalem, Israel, the XXIII AIEL National Conference of Labour Economics, held on September 11th–12th, 2008 at the University of Brescia, Italy, the Department of Economics of the University of Pisa, Italy, for stimulating discussions and very helpful comments and suggestions that have substantially improved the quality of the paper. Usual disclaimers apply.

[*] *E-mail address*: lfanti@ec.unipi.it; tel.: +39 050 22 16 369; fax: +39 050 22 16 384.

[**] Corresponding author. *E-mail address*: luca.gori@ec.unipi.it; tel.: +39 050 22 16 212; fax: +39 050 22 16 384.

1. Introduction

An important and largely debated argument in the economic literature deals with the effects of minimum wages in aggregate macroeconomic models in both static and dynamic contexts. Opponents viewed the minimum wage as a bad social policy, essentially because it deteriorates employment and output.[1] Proponents, instead, typically focused on redistributive goals the minimum wage may create. Moreover, minimum wage legislation can also have important interactions with the social welfare system (e.g., the unemployment benefit system, see Shimer and Werning, 2007), especially in Europe where labour market rigidities represent relevant aspects of real phenomena (see, e.g., Blanchard, 1998).

Even if minimum wages and unemployment compensations cannot probably be expected to greatly reduce household's poverty (see, e.g., OECD, 1998; Müller and Steiner, 2008), their effectiveness in reducing income inequality among households is recognised to be greater, that is minimum wage legislation has been prevalently used for equity reasons thus trading-off with efficiency goals (see, e.g., Tamai, 2009 for a theoretical analysis of the effects of the minimum wage on income inequality.[2] As regards empirical evidence, two important papers that analyse how

[1] The debate about the effects of the minimum wage on employment has seen renewed interest starting from the works by Neumark and Wascher (1995), where it is found evidence of the existence of significant enrolment and employment shifts due to minimum wage increases for young workers, and by Card and Krueger (1994, 1995, 1998), where positive jobs gains rather than losses can, in some circumstances, be obtained. For an analysis of the effects of introducing or removing the minimum wage on employment and the reservation wage, see also the recent paper by Falk et al. (2006). Finally, an interesting review of the empirical literature on the employment effects of the minimum wage is Neumark and Wascher (2006).

[2] In particular, Tamai (2009) used a continuous-time endogenous growth model à la Romer with heterogeneous households (divided by ability) and political determination of the minimum wage to analyse how inequality, unemployment and growth are related. He found that a positive correlation between inequality and unemployment can

institutional changes in the labour market can affect wage inequality in the United States, are DiNardo et al., 1996; Fortin and Lemieux, 1997[3]).

Our knowledge of the effects of minimum wages in aggregate macroeconomic models has been firstly improved by the seminal paper by Stigler (1946). The basic (one-sector, static, partial equilibrium) model of the minimum wage effects on employment and unemployment focuses on a single competitive labour market with homogeneous workers (all covered by the legislated wage), and predicts that minimum wages cause both employment and output reductions.

In dynamic contexts, however, the impact of legislated wage minima on employment, economic growth and welfare is controversial. In particular, it has been shown that if the minimum wage generates some positive externalities it can be growth improving under certain conditions.

Generally speaking, economic growth models with minimum wages, thus linking growth and unemployment, could be divided at least in three categories: (*i*) overlapping generations two sector closed-economy models where the minimum wage in the market for raw labour generates some positive external effects on the accumulation of human capital (Cahuc and Michel, 1996; Ravn and Sørensen, 1999); (*ii*) minimum wage effects in an open economy either with two-period overlapping generations (Irmen and Wigger, 2006) or in infinite horizon continuous-time growth models (Askenazy, 2003); (*iii*) Schumpeterian growth models with labour market imperfections (Aghion and Howitt, 1994; Meckl, 2004). All these papers, however, strongly deviate from the

---

exist. A rise in inequality (which in turn implies a higher minimum wage), therefore, promotes growth if the former is high enough.

[3] The former authors found that the fall in the minimum wage and the de-unionisation process in the U.S. contributed to explain the rise in wage inequality so that institutions “are as important as supply and demand considerations [see Katz and Murphy, 1992] in explaining changes in the U.S. distribution of wages from 1979 to 1988.” (DiNardo et al., 1996, p. 1001). The latter authors analysed the effects institutional changes in the U.S. in the 1980s founding that de-unionisation can explain the rise in wage inequality for men and the minimum wage is important for women. They concluded, however, that “When men and women are considered together, institutions have an even larger impact on inequality.” (Fortin and Lemieux, 1997, p. 75–76).

basic one-sector closed-economy growth model à la Romer with overlapping generations and Cobb-Douglas utility and production functions, which is precisely the framework used here to assess the role the minimum wage can play on economic growth and welfare when unemployment benefits financed with consumption taxes at a balanced budget also exist, and most important, *no* external effects induced by the minimum wage are involved. In a closed economy with no external effects of the minimum wage, conclusions are essentially for a negative role of the minimum wage on growth and welfare as well as a rise in unemployment.

We now briefly review some of the main contributions relating minimum wages, unemployment and growth. As regards models of point (*i*) above (growth-promoting externalities of the minimum wage), Cahuc and Michel (1996), in particular, studied how the minimum wage for unskilled labour affects economic growth and welfare in a two-sector OLG closed economy model with endogenous growth à la Lucas (1988). They assumed that the introduction of a binding minimum wage in the market for unskilled labour causes a positive external effect on the accumulation of human capital because the demand for skilled labour raises and, hence, unskilled workers wish to improve their level of skills in order to avoid unemployment. Because of such an externality, the minimum wage may promote growth and welfare. However, Cahuc and Michel also remarked that in an exogenous (neoclassical) growth context, minimum wages actually increase unemployment and reduce growth (see Cahuc and Michel, 1996, Proposition 1, p. 1469), in line with the traditional negative view of labour market rigidities. Another important paper where the minimum wage is assumed to create growth-promoting externalities in a closed economy is Ravn and Sørensen (1999). Using the same OLG endogenous growth context as in Cahuc and Michel, they assumed that a minimum wage for the unskilled affects labour productivity growth through two sources of accumulation of skills: schooling before entering the labour market and training on the job, showing that the final effect of a rise in the minimum wage on growth is potentially uncertain because it can induce skill formation through schooling and reduce training.

As regards models of point (*ii*), (minimum wage in open economies) an important contribution is Askenazy (2003), who analysed the effects of the minimum wage on growth in an open economy model but assuming, different from Cahuc and Michel and Ravn and Sørensen, a continuous time endogenous growth model à la Ramsey where the minimum wage causes a shift of efforts from the production sector to the R&D sector and thus may stimulate growth through this channel. Another interesting paper in an open economy context is Irmen and Wigger (2006), where, different from all the three previous mentioned papers, no growth-promoting externalities induced by minimum wages are assumed. In particular, they considered an OLG two-country endogenous growth model à la Romer with capital mobility, and showed that a binding minimum wage in the domestic country may stimulate global economic growth, depending on the elasticity of substitution between capital and efficient labour, the output elasticity of efficient labour and the differences between the propensity to save in both the foreign and domestic countries. This in turn implies that with Cobb-Douglas production function and/or uniform propensity to save, minimum wages would be harmful to the global economic growth, thus confirming that in the absence of an externality channel the conventional belief about the growth depressing role of minimum wages holds.

Moreover, for the sake of completeness, it is important to note that another strand of literature exists where unemployment and growth can be positively related owing to the well-known Schumpeterian idea of creative-disruption (i.e., models of point (*iii*) above that were pioneered by Aghion and Howitt, 1994, where the *intrasectoral* allocation of labour within the sector that produces intermediate goods determines the unemployment rate). In this context, an interesting recent paper that treats minimum wages within an innovation-based growth model with three sectors of production is Meckl (2004). Growth and unemployment are ambiguously related and this depends on the sign of the wage differentials between sectors (the final-good sector; the intermediate-good sector; the R&D sector), i.e. the *intersectoral* allocation of labour matters. The higher the wage in the R&D sector, the more likely growth and unemployment will be positively correlated.

As we briefly mentioned above, all these papers abstract from studying the effects of legally set wage minima and unemployment benefits in a simple one-sector closed-economy model with endogenous growth. The object of the present paper, therefore, is to fill this gap using a minimal set of assumptions within a double Cobb-Douglas OLG economy, where the usual technological externality à la Romer (1986) represents the engine of endogenous growth, and, different from the previous theoretical literature framed in the OLG context, without assuming the existence of any growth sustaining externalities the minimum wage can generate. In particular, we study the interaction between minimum wages and growth, showing a new theoretical channel with which minimum wage policies can effectively be used as a stimulus to economic growth and welfare, despite the resulting unemployment they create. In fact, even if the introduction of a wage floor causes unemployment, if the replacement rate (as part of the unemployment benefit system) is larger than the weight of the labour input in production the positive growth effect due to the increased workers' income dominates the negative unemployment effect, and this eventually spurs economic growth beyond the *laissez-faire* level.

In particular, in this paper we provide necessary and sufficient conditions for a regulated-wage economy with unemployment to grow faster than the competitive-wage economy with full employment. Interestingly, there exist (*i*) a whole range of minimum wages that can be used by the government to promote growth, and (*ii*) a growth-maximising minimum wage. We also show that in the long run, individuals may be better off in an economy with regulated-wages and the highest possible welfare level is achieved when growth is maximised. Moreover, an important result should be remarked: the growth and welfare promoting effects of the minimum wage hold in the case of both homogeneous and heterogeneous (i.e., skilled and unskilled) labour. While in the former case, the minimum wage is defined as a mark up over the equilibrium competitive wage and cover all workers, in the latter case it is fixed as a mark up over competitive wage in market for raw labour only. However, since in real economies the minimum wage is computed as a fraction of average earnings, and beneficiaries are essentially the poorest amongst low-paid workers, in the model with labour heterogeneity we also present numerical simulations to test for the robustness of our theoretical results by considering the more realistic case of a minimum wage floor fixed as a percentage of the average wage. As to this purpose, we calibrate our stylised economy to study the case of France, where debates about whether increasing or reducing the minimum wage were intense.

The remainder of the paper is organised as follows. In Section 2 we develop the baseline model with homogeneous labour and consumption taxation not conditioned on age. In Section 3 we analyse the effects of the minimum wage on economic growth. Section 4 looks at welfare effects the minimum wage generates. Section 5 presents some extensions and introduces, in particular, the hypothesis of skilled and unskilled labour with a minimum wage being fixed only for the latter category. Section 6 concludes.

2. The model

2.1. Individuals

Consider a general equilibrium closed economy with overlapping generations. Each generation is composed by a continuum of N identical two-period lived individuals (Diamond, 1965). When young, each individual is endowed with one unit of time inelastically supplied on the labour market. When old, she is retired.

The lifetime (logarithmic) utility function of agent j born at t (U^j_t) is defined over young-aged and old-aged consumptions, $c^j_{1,t}$ and $c^j_{2,t+1}$, respectively, that is:

$$U^j_t = \ln\left(c^j_{1,t}\right) + \beta \ln\left(c^j_{2,t+1}\right), \tag{1}$$

where $0 < \beta < 1$ represents the degree of individual (im)patience to consume over the lifecycle.

Individuals at t can either be employed ($j = e$) or unemployed ($j = u$). If employed, they earn a unitary minimum wage, $w_{m,t}$, fixed by law as a constant mark up $\mu > 1$ over the prevailing competitive wage, $w_{c,t}$, see, e.g., Irmen and Wigger (2006), that is:

$$w_{m,t} := \mu w_{c,t} . \tag{2.1}$$

If unemployed, they receive an unemployment benefit b_t defined as a fraction of the competitive wage,[4] that is:

$$b_t := \gamma w_{c,t} , \tag{2.2}$$

where $0 < \gamma < 1$ is the replacement rate. We assume that unemployment benefits at t are financed with *ad valorem* taxes ($\tau_t > 0$) levied on both young-aged and old-aged consumptions of all people.

Therefore, the budget constraint when young reads as

$$c^j{}_{1,t}(1+\tau_t) + s^j{}_t = x^j{}_t , \tag{3.1}$$

where $s^j{}_t$ is the saving rate of agent j and $x^j{}_t = \{w_{m,t}, b_t\}$ is an income-when-young variable equal to: (*i*) the minimum wage if the individual is employed; (*ii*) the unemployment benefit if she is unemployed.[5]

The budget constraint of an old born at t is

$$c^j{}_{2,t+1}(1+\tau_{t+1}) = (1+r_{t+1}) s^j{}_t , \tag{3.2}$$

where r_{t+1} is the interest rate from t to $t+1$.

[4] Note that the results of the present paper would have been qualitatively the same if unemployment benefits were proportional to the actual minimum wage, rather than the competitive wage. We thank an anonymous referee for suggesting to clarify this point.

[5] By passing we note that the right-hand side of Eq. (3.1), can alternatively be expressed as $d\, w_{m,t} + (1-d) b_t$, where d is a variable that takes value 1 if the individual is employed and 0 is she is unemployed (see Corneo and Marquardt, 2000).

Employed and unemployed individuals choose how much to save out of their disposable income to maximise the lifetime utility function Eq. (1) subject to Eqs. (3). The first order conditions for an interior solution therefore are the following:

$$\frac{c^j{}_{2,t+1}}{c^j{}_{1,t}} = \beta(1+r_{t+1})\frac{1+\tau_t}{1+\tau_{t+1}}. \tag{4}$$

Combining Eq. (4) with Eqs. (3), when the individual j is alternatively employed or unemployed, gives young-aged consumption, old-aged consumption and saving of both the employed and unemployed of generation t, which are respectively given by:

$$c^j{}_{1,t} = \frac{x^j{}_t}{(1+\beta)(1+\tau_t)}, \tag{5.1}$$

$$c^j{}_{2,t+1} = \frac{\beta(1+r_{t+1})x^j{}_t}{(1+\beta)(1+\tau_{t+1})}, \tag{5.2}$$

$$s^j{}_t = \frac{\beta\, x^j{}_t}{1+\beta}. \tag{5.3}$$

Defining L_t as the number of employed people at t, aggregate saving in the economy ($S_t = Ns_t$) is defined as the sum of savings of both employed and unemployed, that is $Ns_t = L_t s^e{}_t + (N - L_t)s^u{}_t$, that can alternatively be written as

$$s_t = (1-u_t)s^e{}_t + u_t s^u{}_t, \tag{6.1}$$

where $u_t := (N - L_t)/N$ is the aggregate unemployment rate at t.[6] Therefore, combination of Eqs. (5.3) and (6.1) yields aggregate savings as:

$$s_t = \frac{\beta}{1+\beta}\left[w_{m,t}(1-u_t)+b_t u_t\right]. \tag{6.2}$$

Using the same line of reasoning, summing up young-aged and old-aged consumptions of both the employed and unemployed from Eqs. (5.1) and (5.2) gives

[6] Eq. (6.1) reveals that separating with employed and unemployed people is the same as assuming a representative individual that can be employed for the fraction $1-u_t$ of time and unemployed for the remaining fraction u_t.

$$c_{1,t} = \frac{1}{(1+\beta)(1+\tau_t)}\left[w_{m,t}(1-u_t)+b_t u_t\right], \tag{7.1}$$

$$c_{2,t+1} = \frac{\beta}{(1+\beta)(1+\tau_{t+1})}(1+r_{t+1})\left[w_{m,t}(1-u_t)+b_t u_t\right]. \tag{7.2}$$

2.2. Firms

As in Romer (1986), Daveri and Tabellini (2000) and Irmen and Wigger (2002), we assume the technology of production faced by each firm $i = 1, \ldots, I$ as:

$$Y_{i,t} = K_{i,t}{}^{\alpha}\left(A_{i,t}L_{i,t}\right)^{1-\alpha} = B k_t^{1-\alpha} K_{i,t}{}^{\alpha} L_{i,t}{}^{1-\alpha}, \tag{8}$$

where $Y_{i,t}$, $K_{i,t}$ and $L_{i,t}$ are the output produced, and capital and labour hired by firm i, $A_{i,t} := a\frac{K_t}{N}$ is an index of labour productivity of each single firm, which is assumed to depend on the average per capita stock of capital in the economy, $k_t = K_t / N$, and it is taken as given by firm i, $B := a^{1-\alpha} > 0$ is a scale parameter and $0 < \alpha < 1$. Since all firms are identical, setting $L_{i,t} = L_t$, $K_{i,t} = K_t$ and $Y_{i,t} = Y_t$, then aggregate production at t is the following $Y_t = B k_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$, where $L_t = (1-u_t)N$ is the total labour force employed. Therefore, per capita output is $y_t = B k_t (1-u_t)^{1-\alpha}$, where $y_t = Y_t / N$, and profit maximisation implies[7]

$$r_t = \alpha B (1-u_t)^{1-\alpha} - 1, \tag{9.1}$$

$$w_{m,t} = (1-\alpha) B k_t (1-u_t)^{-\alpha}. \tag{9.2}$$

Combining Eqs. (2.1) and (9.2), and knowing that $w_{c,t} = (1-\alpha) B k_t$ is the equilibrium competitive wage, the (constant) unemployment rate in this simple economy with homogeneous labour is

[7] It is assumed that capital totally depreciates at the end of each period and output is sold at unit price.

$$u_t = u(\mu) = 1 - \mu^{\frac{-1}{\alpha}}. \tag{10.1}$$

From Eq. (10) it is easy to verify that a rise in μ (γ) monotonically increases (does not affect) unemployment. Moreover, the interest rate is lower than under *laissez-faire*, that is:

$$r(\mu) = \alpha B \mu^{\frac{\alpha-1}{\alpha}} - 1 < r(1).^{8} \tag{10.2}$$

2.3. Government

The unemployment benefit expenditure at t ($b_t u_t$) is financed with (time-adjusted)[9] *ad valorem* taxes levied on the first and second period consumptions of *all* individuals. Therefore, at time t the per capita government budget reads as:

$$b_t u_t = \tau_t (c_{1,t} + c_{2,t}). \tag{11}$$

Exploiting Eqs. (2.1), (2.2), (7.1), the one-period backward (7.2),[10] (9.1) and (10.1), the budget-balancing tax rate is constant and given by:

$$\tau_t = \tau(\mu) = \frac{\gamma(1-\alpha)(1+\beta)\left(\mu^{\frac{1}{\alpha}} - 1\right)}{\mu(1+\alpha\beta) - \gamma(1-\alpha)\beta\left(\mu^{\frac{1}{\alpha}} - 1\right)}, \tag{12}$$

---

[8] Note that $r(\mu) > 0$ for any $\mu < (\alpha B)^{\frac{\alpha}{1-\alpha}} := \mu_\Omega$, which is assumed to be always satisfied in the analytical results of the present paper. Moreover, in the numerical examples presented in Sections 3–5 the interest rate associated with different values of the wage mark up used in the simulations is of course always positive.

[9] In Section 5.1 we study the opposite case of endogenous replacement rate and fixed consumption tax rate.

[10] Since both the minimum wage and unemployment benefit are introduced at the beginning of time t, the consumption of the old-aged born at $t-1$ is $c_{2,t} = \frac{\beta(1+r_t)w_{c,t-1}}{(1+\beta)(1+\tau_t)}$, where $w_{c,t-1} = \frac{w_{c,t}}{1+g_c}$ and $g_c = \frac{\beta}{1+\beta}(1-\alpha)B - 1$ is the growth rate of per capita income under *laissez-faire*.

where $\tau(1)=0$ and $\tau'(\mu)>0$. Now, in order to make the unemployment benefit policy feasible, we need to ensure $\tau>0$ for any couple (μ,γ), i.e. denominator of Eq. (12) must be positive. Therefore, for any $0<\gamma<1$, the benefit expenditure is viable if and only if the government does not fix the minimum wage at too high a level, that is $1<\mu<\mu_{\mathrm{M}}$, where μ_{M} (that cannot be solved in closed form, however) represents the threshold value of the wage mark up below which $\tau>0$ is fulfilled.

3. Balanced growth

Given the government budget Eq. (11), market-clearing in goods and capital markets is expressed as

$$k_{t+1}=s_t\,, \tag{13}$$

which is combined with Eq. (6.2) to get:

$$k_{t+1}=\frac{\beta}{1+\beta}\left[w_{m,t}(1-u_t)+b_t\,u_t\right]. \tag{14}$$

Since in this paper we are mainly interested in studying the effects of minimum wages on economic growth, below we analyse how legislated wage minima affect the evolution of capital across periods and, hence, the growth rate of the economy. To this purpose, let us first rewrite Eq. (14) as a generic function of the wage mark up as

$$k_{t+1}=s\left[w_{m,t}(\mu),u(\mu)\right]. \tag{15}$$

The total derivative of Eq. (15) with respect to μ gives:

$$\frac{dk_{t+1}}{d\mu}=\underbrace{\overbrace{\frac{\partial s}{\partial w_{m,t}}}^{+}\cdot\overbrace{\frac{\partial w_{m,t}}{\partial\mu}}^{+}\cdot\overbrace{(1-u)}^{+}}_{+}+\underbrace{\overbrace{\frac{\partial s}{\partial u}}^{-}\cdot\overbrace{\frac{\partial u}{\partial\mu}}^{+}\cdot\overbrace{(w_{m,t}-b_t)}^{+}}_{-}, \tag{16}$$

Eq. (16) reveals that the final effect of a rise in the minimum wage (i.e., a rise in the wage mark up) is ambiguous on growth as it increases both wage income and unemployment. The economic intuition is simple. Since in an OLG context economic growth is driven by savings, it will be crucial how the minimum wage affects the saving rate. On the one hand, a rise in the minimum wage causes a positive effect on savings because the income of the employed is now larger. On the other hand, the unemployment rate raises. The rise in unemployment affects aggregate savings through two channels of opposite sign: first, the amount of resources saved by the employed is now lower because the employment rate is reduced, so that the saving rate shrinks through this channel; second, the amount of resources saved by the unemployed is now larger, and this, in turn, positively affects savings. However, since the minimum wage is larger than the unemployment benefit, the rise in unemployment always tends to reduce aggregate savings, as can easily be ascertained from the second addendum of the right-hand side of Eq. (16).

In sum, there exists both a positive wage effect and negative unemployment effect on growth when μ raises, and this makes clear the reason why the final effect of a rise in the minimum wage on capital accumulation is potentially uncertain in an OLG model.[11]

We now exploit Eqs. (2.1), (2.2), (10.1) and (14) to determine how capital evolves across periods, that is:

$$k_{t+1} = (1+g_c)\mathrm{H}(\mu)k_t\,, \tag{17}$$

where $\mathrm{H}(\mu) := \mu^{\frac{-1}{\alpha}}\left[\mu + \gamma\left(\mu^{\frac{1}{\alpha}} - 1\right)\right]$, $\mathrm{H}(1) = 1$ and g_c is the growth rate under *laissez-faire* (see Footnote 7).

[11] Notice that in a Ramsey-type infinite horizon growth model, where economic growth is driven by the interest rate rather than the saving rate, minimum wages (even if used together with unemployment benefit policies) reduce the growth rate of consumption when the technological externality is defined in per capita terms, because the interest rate is below the *laissez-faire* level due to the unemployment occurrence, as Eq. (10.2) reveals (see, e.g., Hellwig and Irmen, 2001). However, if we alternatively assume, in line with Corneo and Marquardt (2000), that the production externality is defined in terms of capital per employed worker (see Ono, 2007 for a discussion about the differences in assuming that labour efficiency depends on either capital per capita or capital per worker), then the introduction of minimum wages is growth-neutral in a growth model à la Ramsey because the interest rate does not depend on the unemployment rate in that case.

From Eq. (17), therefore, the growth rate of capital in the regulated-wage economy with unemployment (which is the same as the growth rate of per capita output as the unemployment rate is constant) is:[12]

$$g(\mu) = (1 + g_c)\mathrm{H}(\mu) - 1. \tag{18}$$

Now, let

$$\bar{\gamma} := 1 - \alpha , \tag{19}$$

be a threshold value of the replacement rate. Then, analysis of Eq. (18) gives the following proposition.

Proposition 1. *Let* $1 < \mu < \mu_{\mathrm{M}}$ *hold to guarantee feasibility of the unemployment benefit policy. (1) If* $\gamma \le \bar{\gamma}$*, then* $g(\mu) < g_c$*. (2) If* $\gamma > \bar{\gamma}$*, then* $g(\mu) > g_c$ *for any* $1 < \mu < \mu^{\circ}$*,* $g(\mu)$ *is maximised at* $\mu = \hat{\mu}$ *and* $g(\mu) < g_c$ *if* $\mu > \mu^{\circ}$*, where*

$$\hat{\mu} := \frac{\gamma}{\bar{\gamma}} > 1, \tag{20}$$

and $\mu^{\circ} > \hat{\mu}$ *is the value of the wage mark up such that* $g(\mu^{\circ}) = g_c$.

Proof. Differentiating Eq. (18) with respect to μ gives $g'(\mu) = (1 + g_c)\mathrm{H}'(\mu)$, where $\mathrm{H}'(\mu) = \alpha^{-1}\mu^{\frac{-(1+\alpha)}{\alpha}}(\gamma - \mu\bar{\gamma})$ and thus $\mathrm{sgn}\{g'(\mu)\} = \mathrm{sgn}\{\mathrm{H}'(\mu)\}$. Then $\mathrm{H}'(\mu) \gtreqless 0$ if $\mu \lesseqgtr \hat{\mu}$. Therefore Proposition 1 follows, since (1) if $\gamma \le \bar{\gamma}$, $\mathrm{H}'(\mu) < 0$ for any $1 < \mu < \mu_{\mathrm{M}}$, and (2) if $\gamma > \bar{\gamma}$, the fact that $\mathrm{H}'(\mu) = 0$ only at $\mu = \hat{\mu}$ and $\mathrm{H}''(\mu) = 0$ only at $\mu = \hat{\mu}(1 + \alpha) > \hat{\mu}$ complete the proof because $\mathrm{H}(\mu) = 1$ twice at $\mu = 1$ and $\mu = \mu^{\circ}$. **Q.E.D.**

[12] Since $g(\mu)$ is independent of time the model does not show transitional dynamics, and thus a change in μ automatically results in an instantaneous adjustment to a new balanced growth path.

Corollary 1. *Let* $\mu = 1$ *hold. If* $\gamma < \bar{\gamma}$ $(\gamma > \bar{\gamma})$ *then the introduction of minimum wages reduces (promotes) economic growth.*

Proof. The proof is straightforward since $g'(1) < 0$ (> 0) for any $0 < \gamma < \bar{\gamma}$ ($\bar{\gamma} < \gamma < 1$). **Q.E.D.**

Proposition 1 shows that the growth rate in a regulated-wage economy with unemployment may be larger than under *laissez-faire*. This is due to a combination of minimum wages and unemployment benefits. In fact, although the minimum wage causes unemployment, if the replacement rate is larger than the weight of the labour input in production (i.e., $1-\alpha$), the positive growth effect due to the increased workers' income dominates the negative unemployment effect, and this eventually spurs economic growth beyond the *laissez-faire* level. Moreover, it is important to note that (*i*) a large enough replacement rate always exists to guarantee beneficial effects of minimum wages on growth, (*ii*) the higher the output elasticity of capital, the lower the size of the replacement rate needed for introducing a growth-promoting minimum wage policy, and (*iii*) a growth-maximising minimum wage can exist. Moreover, since both the growth rate and unemployment rate increase along with the wage mark up, at least for any $1 < \mu < \hat{\mu}$, then a positive link between unemployment and growth indirectly exists in such a case. Therefore, in a broad sense, our paper may be view as belonging to the literature linking positively economic growth and unemployment (see the discussion in Section 1), but using however simple ingredients within an intuitive context such as the two-period overlapping generations growth model rather than adopting questionable hypotheses within more complicated settings.

Of course the way of financing the benefit expenditure is crucial for the results. In fact, with consumption taxes, the provision of unemployment benefits is not completely retrieved by the amount of resources needed to finance it (as, instead, would be the case with either wage income taxes or lump-sum taxes on the young, which would imply both a tax withdrawal and unemployment subsidy of the same size, thus leaving the aggregate income of the young unaffected), so that the unemployed will actually receive a grant – and the employed are burdened by taxation – in such a way that aggregate consumption, saving and growth are effectively supported despite the rise in unemployment.

We now illustrate Proposition 1 taking the following configuration of technological parameters: $\alpha = 0.45$,[13] which generates $\bar{\gamma} = 0.55$, and $B = 20$. Then we choose a replacement rate higher than the threshold $\bar{\gamma}$, that is $\gamma = 0.7$ (in line with the unemployment benefit legislation in several European countries). Therefore, the growth-maximising wage mark up is $\hat{\mu} = 1.272$ and $\mu^{\circ} = 1.778$. As regards preferences, we consider that every period consists of 30 years and assume $\beta = 0.3$ so that the discount factor is 0.96 per annum (see de la Croix and Michel, 2002, p. 50).[14] With this parameter values we obtain $g(\hat{\mu}) = 1.627$, corresponding to which the unemployment rate is $u(\hat{\mu}) = 0.414$ and the budget-balancing consumption tax rate is $\tau(\hat{\mu}) = 0.26$, while the growth rate under *laissez-faire* is $g_c = 1.538$. Moreover, a whole range of values of the wage mark up, i.e. $1 < \mu < 1.778$, can be used by the government to promote growth. In this example, the unemployment rate that corresponds to the growth-maximising minimum wage (which is 27.2 per cent higher than the competitive wage) is extremely high (41.4 per cent), and the range of minimum wages for which an economy grows at a rate higher than the laissez-faire may even reach up the 77.8 over the market wage. In this stylised economy, however, labour is homogeneous (rather than heterogeneous) and the minimum wage is set as a mark up over the market wage (rather than computed as a fraction of average earnings, as in several real economies). Therefore, in order to evaluate our results for drawing policy conclusions, it may be relevant to introduce labour heterogeneity (e.g., skilled and unskilled labour) and then computing the minimum wage as a

[13] This value may be considered as an average between the values of the output elasticity of capital in developed and developing countries, which, according to, e.g., Kraay and Raddatz (2007), are $\alpha = 0.33$ and $\alpha = 0.5$, respectively.

[14] Note that with this parameter set we obtain a large enough value of the wage mark below which feasibility of the unemployment benefit policy is guaranteed, that is $\mu_{\mathrm{M}} = 6.568$.

percentage of the average wage. This exercise will be presented in Section 5.2 after showing that considering skilled (high-paid) and unskilled (low-paid) labour does not alter the main conclusions of Proposition 1 as regards the effects on growth of introducing the minimum wage in the market for raw labour.

While we have shown that economic growth can actually be fostered by minimum wages, another crucial aspect regards their welfare effects. The next section therefore deals with this argument and contrasts welfare levels in both the regulated-wage economy with unemployment and competitive-wage economy with full employment.

4. Welfare

This section looks at the evolution of individual welfare over time in both the regulated-wage and competitive-wage economies along the balanced growth path (BGP henceforth).

Although the introduction of wage minima cannot represent a Pareto improvement, because the old-aged living at the moment of the reform would not opt for it since (*i*) the interest rate shrinks due to the unemployment occurrence (see, Eq. 10.2), and (*ii*) the existence of consumption taxes to finance unemployment benefits tends to deteriorate their consumption level,[15] it could be instructive studying whether the minimum wage can make individuals better off along the BGP. Below we show that when $g(\mu) > g_c$ (i.e., Point (2) of Proposition 1 holds), then, asymptotically, the minimum wage is always welfare-improving.

---

[15] Let the minimum wage be introduced at t. Then, comparison of the consumption of the old-aged born at $t-1$ in both the regulated-wage and competitive-wage economies implies $c_{2,t}(\mu) < c_{2,t}(1)$, where $c_{2,t}(\mu) = \frac{\beta[1+r(\mu)]w_{c,t-1}}{(1+\beta)[1+\tau(\mu)]}$, $c_{2,t}(1) = \frac{\beta[1+r(1)]w_{c,t-1}}{1+\beta}$, $r(\mu) < r(1)$ and $\tau(\mu) > 0$.

Let us begin the welfare analysis by noting that applying a positive monotonic transformation, the individual lifetime utility function Eq. (1) can also be expressed as $V_t = (c_{1,t})^{\frac{1}{1+\beta}}(c_{2,t+1})^{\frac{\beta}{1+\beta}}$, where $V_t := e^{\frac{U_t}{1+\beta}}$. Since both young-aged and old-aged consumption grow without transition at the constant rate $g(\mu)$,[16] the time evolution of individual welfare in an economy with legislated wage minima is expressed by the indirect utility function:

$$W_t(\mu) = W_0(\mu)[1+g(\mu)]^t \,, \tag{21}$$

where

$$W_0(\mu) = W_0(1)\Pi(\mu), \tag{22}$$

is the welfare level at the time of the introduction of the minimum wage law, i.e. $t=0$,

$$\Pi(\mu) := \frac{\mathrm{H}(\mu)}{1+\tau(\mu)}\mu^{\frac{(\alpha-1)\beta}{\alpha(1+\beta)}} = \frac{\mu^{\frac{1+\beta(2-\alpha)}{\alpha(1+\beta)}}\left[\mu+\gamma\left(\mu^{\frac{1}{\alpha}}-1\right)\right]\left[\mu(1+\alpha\beta)-\gamma(1-\alpha)\beta\left(\mu^{\frac{1}{\alpha}}-1\right)\right]}{\mu(1+\alpha\beta)+\gamma(1-\alpha)\left(\mu^{\frac{1}{\alpha}}-1\right)}, \tag{23}$$

which crucially depends on the level of the existing minimum wage, and

$$c_{1,0}(1) = \frac{(1-\alpha)Bk_0}{1+\beta}\,, \tag{24.1}$$

$$c_{2,1}(1) = \frac{\beta\alpha(1-\alpha)B^2k_0}{1+\beta}\,, \tag{24.2}$$

$$W_0(1) = [c_{1,0}(1)]^{\frac{1}{1+\beta}}[c_{2,1}(1)]^{\frac{\beta}{1+\beta}}\,, \tag{22'}$$

are the initial consumption and welfare levels under *laissez-faire*, with $k_0 > 0$ given.

The time evolution of individual welfare in an economy with full employment therefore can be conveyed by:

$$W_t(1) = W_0(1)(1+g_c)^t\,. \tag{21'}$$

[16] This follows because the unemployment rate, the interest rate and the budget-balancing consumption tax rate are constant.

From Eqs. (21) and (21') it is clear that BGP welfare generically depends on (*i*) the growth rate of per capita income, and (*ii*) the initial welfare level (i.e. initial consumption). As regards the former, to the extent that a rise in the wage mark up is beneficial to economic growth, i.e. Point (2) of Proposition 1 is fulfilled, the effects of minimum wages on welfare are positive through this channel. As regards the latter, comparison of Eqs. (22) and (22') makes clear that individual welfare at the time of the introduction of the minimum wage law, $t=0$, can be higher or lower than under *laissez-faire* depending on whether $\Pi(\mu)$ is higher or lower than unity. A rise in the wage mark up increases the income of the young, and this tends to raise initial consumption. However, a higher minimum wage also implies a higher tax rate to finance an increasing amount of unemployment benefits, and the higher the minimum wage, the higher the consumption tax rate. This, in turn, tends to deteriorate initial consumption.

Therefore, if $\Pi(\mu)<1$ (that is, consumption at $t=0$ in the regulated-wage economy is lower than under *laissez-faire*, because the weight of consumption taxation is relatively high), then the final effect of a rise in the wage mark up μ on welfare is *a priori* uncertain because of the existence of two opposite forces at work: (*i*) the positive growth effect, and (*ii*) the negative effect due to the deteriorated consumption at $t=0$. In this case, therefore, the minimum wage causes welfare losses for the current old-aged, for the generation born at time $t=0$ and for some generations $t>T>0$. However, we will show below that beyond the threshold generation T the minimum wage implies welfare gains because the positive growth effects always dominates in the long run, i.e. when Point (2) of Proposition 1 holds consumption grows at a rate higher than under *laissez-faire* and, thus, there certainly exists a threshold generation beyond which welfare becomes larger despite the rise in unemployment.

In contrast, if $\Pi(\mu)>1$ (that is, consumption at $t=0$ in the regulated-wage economy is larger than under *laissez-faire* because the weight of consumption taxation is relatively low), then we obtain the important result that the generation born at $t=0$ as well as *all* the (infinite) subsequent generations will be better off in that case. The minimum wage policy, however, cannot be Pareto optimal because the current old-aged are worsened.

We now proceed to show the results discussed above. First, using Eq. (23) we identify the parametric conditions for which $\Pi(\mu)>1\,(<1)$. Second, since the analytical treatment of Eq. (21) is cumbersome, we resort to numerical simulations to show that in the long run: (*i*) the minimum wage can effectively produce welfare gains either for all generations $t \geq 0$ or for all generations t born beyond the threshold $T > 0$, and (*ii*) in the long run the highest possible welfare level is achieved when economic growth is maximised.

We now identify the conditions for which the introduction of the minimum wage evaluated at the margin ($\mu = 1$) may or may not be welfare-improving for all generations $t \geq 0$ but the current old-aged, i.e. the ones born at $t = -1$.

Define $F(\mu) := \Pi(\mu) - 1$, $\alpha_W := \dfrac{\beta}{1+2\beta} < \dfrac{1}{3}$ and $\gamma_W := \dfrac{(1-\alpha)(1+\alpha\beta)(1+2\beta)}{(1+\beta)[\alpha(1+2\beta)-\beta]}$, with $0 < \gamma_W < 1$ if and only if $\alpha < \alpha_{W,1}$ or $\alpha > \alpha_{W,2}$, where

$$\alpha_{W,1} := \frac{-(1+2\beta) - \sqrt{2\beta^4 + 7\beta^3 + 9\beta^2 + 5\beta + 1}}{\beta(1+2\beta)} < 0,$$

$$\alpha_{W,2} := \frac{-(1+2\beta) + \sqrt{2\beta^4 + 7\beta^3 + 9\beta^2 + 5\beta + 1}}{\beta(1+2\beta)}, \quad 1/2 < \alpha_{W,2} < 1.$$

Since $\alpha_{W,1} < 0$ it can be ruled out. Then we have the following proposition.

Proposition 2. *(1) Let* $0 < \alpha < \alpha_W$ *hold. Then, the introduction of minimum wages (evaluated at* $\mu = 1$*) is welfare-worsening at* $t = 0$. *(2.1) Let* $\alpha_W < \alpha < \alpha_{W,2}$ *hold. Then* $\gamma_W > 1$ *and for any* $0 < \gamma < 1$ *the introduction of minimum wages (evaluated at* $\mu = 1$*) is welfare-worsening at* $t = 0$. *(2.2) Let* $\alpha_{W,2} < \alpha < 1$ *hold. Then for any* $0 < \gamma < \gamma_W$ *[*$\gamma_W < \gamma < 1$*] the introduction of minimum wages (evaluated at* $\mu = 1$*) is welfare-worsening [welfare-improving] at* $t = 0$.

Proof. Differentiating $F(\mu)$ with respect to μ and evaluating it at $\mu = 1$ gives

$$F'_\mu(1) = \frac{(1+\beta)[\alpha(1+2\beta)-\beta] - \gamma(1-\alpha)(1+\alpha\beta)(1+2\beta)}{\alpha(1+\beta)(1+\alpha\beta)^2}.$$

Therefore, if $0 < \alpha < \alpha_W$, then $F'_\mu(1) < 0$. In contrast, if $\alpha_W < \alpha < \alpha_{W,2}$, then since $\gamma_W > 1$, $F'_\mu(1) < 0$; if $\alpha_{W,2} < \alpha < 1$, then for any $0 < \gamma < \gamma_W$ [$\gamma_W < \gamma < 1$], $F'_\mu(1) < 0$ [$F'_\mu(1) > 0$]. **Q.E.D.**

Proposition 2 reveals that the lower is the weight of employed labour in production, and the higher is the replacement rate, the more likely the introduction of the minimum wage increases consumption at $t = 0$. In this case, in fact, the weight of the increased budget-balancing tax rate on consumption is relatively low with respect to the increased income when young, and thus welfare raises even at $t = 0$.

We now consider how a rise in the minimum wage above the existing level ($\mu > 1$) affects the function Eq. (23) in the case $\Pi(\mu) > 1$. Although the analysis of Eq. (23) is cumbersome, it can be shown that, in general, $\Pi(\mu)$ is an inverted U-shaped function of the wage mark up μ. The economic reason is that there exists an interval included between the competitive wage (i.e. $\mu = 1$) and a threshold value of the minimum wage (i.e. $\mu = \mu_\Pi$, where μ_Π cannot be solved in closed form) such that all generations $t \geq 0$ can experience welfare gains (of course if Point (2.2) of Proposition 2 holds). This also means that if the wage mark up is fixed beyond the threshold μ_Π, some initial generations $0 \leq t < T$ are harmed. An illustration of this finding is provided in Table 4.

We now assume that the conditions of Point (2) of Proposition 1 hold and proceed with some numerical exercises to analyse the welfare effects of μ over time. First, we concentrate on the case $\Pi(\mu) < 1$. Then we study the case $\Pi(\mu) > 1$. This because the want to stress that there exist cases with respect to which the minimum wage can be welfare improving for either all the generations $t \geq 0$ but the current old-aged or for all the generations t born beyond a threshold generation $T > 0$, and then to precisely identify such a threshold generation in the case some initial generations are harmed by the minimum wage policy.

Using the same parameter set as in Section 3,[17] Table 2 summarises the time evolution of the individual welfare for the values of the wage mark up reported in Table 1 (Column 1), which also shows the corresponding values of the unemployment rate, the budget-balancing consumption tax rate and the growth rate (Columns 2, 3 and 4, respectively).

Table 1. The wage mark up and other macroeconomic and policy variables.

μ	$u(\mu)$	$\tau(\mu)$	$g(\mu)$
1	0	0	1.538
1.1	0.19	0.096	1.598
1.272	0.414	0.26	1.627
1.4	0.526	0.381	1.618

Table 2. The time evolution of individual welfare in both the regulated-wage and competitive-wage economies when μ varies ($k_0 = 1$, $W_t / 10000$).

	$t=0$	$t=1$	$t=3$	$t=5$	$t=7$	$t=9$
$W_t(1)$	0.001	0.0027	0.017	0.112	0.722	4.657
$W_t(1.1)$	0.0009	0.0025	0.016	0.114	0.773	5.222
$W_t(1.272)$	0.0008	0.0021	0.0148	0.102	0.706	4.875
$W_t(1.4)$	0.0007	0.0018	0.0129	0.088	0.609	4.178

[17] Note that with this configuration of parameters we get $\alpha_{W,2} = 0.572$ and, hence, the condition $\gamma_W > 1$ is satisfied (see Point (2.1) of Proposition 2). Since $\alpha = 0.45 < 0.572$, then the minimum wage is welfare-worsening at $t = 0$ (see Table 2, Column 1).

	$t=11$	$t=13$	$t=15$	$t=18$	$t=30$	$t=50$
$W_t(1)$	30.01	193.38	1246.10	20382.87	$1.45\cdot 10^{9}$	$1.80\cdot 10^{17}$
$W_t(1.1)$	35.26	238.11	1607.78	28209.37	$2.67\cdot 10^{9}$	$5.26\cdot 10^{17}$
$W_t(1.272)$	33.65	232.40	1604.57	29110.33	$3.15\cdot 10^{9}$	$7.76\cdot 10^{17}$
$W_t(1.4)$	28.64	196.34	1345.94	24156.56	$2.50\cdot 10^{9}$	$5.74\cdot 10^{17}$

Since the budget-balancing tax rate is higher the larger is the wage mark up (see Table 1), the negative welfare effect due to the reduction in initial consumption is stronger the more μ is increased. Thus, Table 2 shows that the generations born at $t=0$ as well as in the subsequent periods $0<t<T$ are made worse off, i.e. the minimum wage policy cannot represent a Pareto improvement. However, in the long run welfare gains are obtained irrespective of the size of the minimum wage, that is a threshold generation $T>0$ exists beyond which all the future generations $t>T$ are made better off because the positive growth effect asymptotically dominates. In fact, from Table 2 we see that when $\mu=1.1$ (1.272) [1.4] individuals of the fifth (ninth) [thirteenth] generation, as well as those born in all future periods, are better off in the regulated-wage economy with unemployment than under *laissez-faire*. Moreover, and most important, Table 2 also shows that the highest possible long-run welfare levels are achieved at the growth-maximising wage mark up.[18]

We now take a value of the output elasticity of capital higher than the threshold $\alpha_{W,2}=0.572$, that is $\alpha=0.7$,[19] to show that the minimum wage can actually be welfare-improving for all the

[18] This has been ascertained through extensive numerical simulations for $t>50$ not reported in Table 2 for economy of space. Moreover, it can be shown (numerically) that the growth-maximising wage mark up coincides with the welfare-maximising one when t becomes large enough.

[19] The other parameter values are the same as before.

generations $t \geq 0$. In this case, therefore, we get $\bar{\gamma} = 0.3$, $\gamma_W = 0.544$, $\hat{\mu} = 2.333$ (the growth-maximising wage mark up), $\mu^{\circ} = 14.82$ and $\mu_{\mathrm{M}} = 988$. Moreover, a whole range of values of the wage mark up, i.e. $1 < \mu < 1.341$, exists such that $\Pi(\mu) > 1$, and then $W_0(\mu) > W_0(1)$ in such a case.

In Table 4 we summarise the time evolution of individual welfare for the values of the wage mark up reported in Table 3 (Column 1).

Table 3. The wage mark up and other macroeconomic and policy variables.

μ	$u(\mu)$	$\tau(\mu)$	$g(\mu)$
1	0	0	0.384
1.1	0.127	0.03	0.452
2.333	0.701	0.24	0.643
3	0.791	0.306	0.632

Table 4. The time evolution of individual welfare in both the regulated-wage and competitive-wage economies when μ varies ($k_0 = 1$, $W_t / 100$).

	$t = 0$	$t = 1$	$t = 3$	$t = 5$	$t = 7$	$t = 9$
$W_t(1)$	0.0642	0.0889	0.17	0.327	0.627	1.202
$W_t(1.1)$	0.0648	0.0941	0.198	0.419	0.884	1.866
$W_t(2.333)$	0.0565	0.0929	0.25	0.677	1.83	4.943
$W_t(3)$	0.052	0.0849	0.226	0.602	1.605	4.276

	$t = 11$	$t = 13$	$t = 15$	$t = 18$	$t = 30$	$t = 50$
$W_t(1)$	2.304	4.418	8.471	22.48	1116	$7.49 \cdot 10^5$

$W_t(1.1)$	3.938	8.311	17.53	53.74	4742	$8.29 \cdot 10^6$
$W_t(2.333)$	13.34	36.05	97.35	432	$1.67 \cdot 10^5$	$3.45 \cdot 10^9$
$W_t(3)$	11.39	30.34	80.83	351.4	$1.25 \cdot 10^5$	$2.26 \cdot 10^9$

Table 4 shows that when the output elasticity of capital is high enough, i.e. $\alpha_{W,2} < \alpha < 1$, and the wage mark up is not fixed at too high a level, i.e. $1 < \mu < \mu_\Pi$, where $\mu_\Pi = 1.341$, the minimum wage is welfare-improving for all generations $t \geq 0$. In this case, in fact, the weight of the increased income when young more than counterbalances the negative effect on consumption due to the increase in the budget-balancing tax rate so that welfare raises. Of course, when the wage mark up increases further on, the weight of the higher tax rate becomes larger and, hence, for any $\mu > \mu_\Pi$ there exists some generations $0 \leq t < T$ that incur in welfare losses because consumption shrinks. To this purpose, Table 4 shows that when $\mu = 2.333$ (3) the positive growth effect dominates at $t = 1$ (3) and welfare gains can then effectively be obtained. Moreover, in the long run the highest possible welfare level is still achieved when the government maximises growth.

To sum up, the following result holds as regards the welfare effects of the minimum wage:

Result 1. *Rewrite Eq. (21) as* $W_t(\mu) = W_t(1)\Pi(\mu)\left[\dfrac{1+g(\mu)}{1+g_c}\right]^t$. *We know from Proposition 1 that if* $\gamma > \bar{\gamma}$, *then* $g(\mu) > g_c$ *for any* $1 < \mu < \mu^\circ$. *Then, for any* $1 < \mu < \mu^\circ$, $\lim_{t \to +\infty} W_t(\mu) = \lim_{t \to +\infty} W_t(1)$. *In this case, therefore, there certainly exists a threshold generation* $T > 0$ *such that* $W_t(\mu) > W_t(1)$ *for any* $t > T$. *Moreover, if Point (2) of Proposition 1 holds,* $\alpha_{W,2} < \alpha < 1$ *and* $\gamma_W < \gamma < 1$, *then for any* $1 < \mu < \mu_\Pi$ *we get* $W_t(\mu) > W_t(1)$ *for any* $t \geq 0$.

In the numerical example presented in Tables 1–4 we studied the cases of a *once-and-for-all* minimum wage policy, i.e. we assumed that at time $t = 0$ a binding minimum wage is introduced, for instance at the growth-maximising value, and then kept unchanged in all future periods. The latter policy, of course, tends to drastically reduce consumption (and welfare) of the first T generations due to the financing of the unemployment benefit with proportional consumption taxes, as the unemployment rate associated with the growth-maximising wage mark up is relatively high. Nevertheless, another important issue is whether welfare losses can actually be smoothed across the first T generations with a *gradual or progressive* increase in the minimum wage from period to period up to the growth-maximising level.[20] Does a gradual introduction in the minimum wage really tend to smooth its impact across generations? In order to answer this question in Table 5 we summarise – using the same parameter set as in Tables 1–2 – the welfare effects of two different minimum wage policies: (*i*) the once-and-for-all growth-maximising policy (Table 5.A), and (*ii*) a progressive increase in the wage mark up to the growth-maximising level (Table 5.B).

Table 5.A. The time evolution of individual welfare in both the regulated-wage and competitive-wage economies: the case of the once-and-for-all minimum wage policy ($k_0 = 1$, $W_t / 10000$).

	$t = 0$	$t = 1$	$t = 3$	$t = 5$	$t = 7$	$t = 9$
$W_t(1)$	0.001	0.0027	0.017	0.112	0.722	4.657
$W_t(1.272)$	0.0008	0.0021	0.0148	0.102	0.706	4.875

[20] This argument is relevant especially because the introduction of the minimum wage in several European countries followed such design, even when the process was rapid as recently in the U.K. or Ireland (see, e.g., Dolado et al, 1996 for evidence of the impact of different minimum wage policies in Europe). We thank an anonymous referee for suggesting to clarify this point.

Table 5.B. The time evolution of individual welfare in both the regulated-wage and competitive-wage economies: the case of a progressive increase in the minimum wage ($k_0 = 1$, $W_t / 10000$).

	$t=0$ $\mu=1.05$	$t=1$ $\mu=1.1$	$t=3$ $\mu=1.15$	$t=5$ $\mu=1.2$	$t=7$ $\mu=1.25$	$t=9$ $\mu=1.272$
$W_t(1)$	0.00106	0.0027	0.017	0.112	0.722	4.657
$W_t(\mu)$	0.00101	0.0025	0.016	0.108	0.721	4.875

Comparison of Tables 5.A and 5.B, therefore, makes clear that it is better to gradually introduce the minimum wage rather than fixing it at the growth-maximising value directly at the time of the introduction of the reform, i.e. $t = 0$, as the welfare losses of the first seven generations are smaller than in the case of the once-and-for-all growth-maximising policy. Moreover, and most important, welfare gains are obtained starting from the ninth generation in both cases. The economic reason why welfare losses are smoothed across generations is that a with progressive increase in the wage mark up, the rise in unemployment is gradual and then lower from period to period. Hence, the need of raising consumption taxes to finance the benefit system as well as the negative impact of the minimum wage on consumption of the initial generations $T > 0$, can actually be smaller in such a case.

5. Extensions

We now present some modifications and extensions of the baseline model. In particular, in Section 5.1 we assume the replacement rate, rather than consumption tax rate, as the endogenous variable that balances the government budget. In Section 5.2 we relax the hypothesis of homogeneous workers and assume the existence of skilled and unskilled labour and then introduce the minimum wage for the unskilled (low-paid) jobs. Finally, in Section 5.3 a capital income tax is used to finance the unemployment benefit system.[21]

5.1. Endogenous replacement rate

In this section we assume the replacement rate γ_t, rather than the consumption tax rate τ, is endogenous and adjusted from period to period to balance out the unemployment benefit budget Eq. (11).

Exploiting Eqs. (2.1), (2.2), (7.1), the one-period backward (7.2), (9.1), (10.1) and (11), the (constant) budget-balancing replacement rate is obtained as

$$\gamma_t = \gamma(\mu,\tau) = \frac{\tau\mu(1+\alpha\beta)}{\left(\mu^{\frac{1}{\alpha}}-1\right)[1+\beta(1+\tau)](1-\alpha)}, \tag{25}$$

where $\gamma'_\mu(\mu,\tau)<0$ and $\gamma'_\tau(\mu,\tau)>0$. Now, the unemployment benefit policy is viable if $\gamma(\mu,\tau)<1$, and this would alternatively require

$$\tau < \hat{\tau} := \frac{(1-\alpha)(1+\beta)\left(\mu^{\frac{1}{\alpha}}-1\right)}{\mu(1+\alpha\beta)-(1-\alpha)\beta\left(\mu^{\frac{1}{\alpha}}-1\right)}. \tag{26}$$

Exploiting Eqs. (2.1), (2.2), (10.1), (14) and using (25) to eliminate the replacement rate, the growth rate in the regulated-wage economy is

$$g(\mu,\tau) = (1+g_c)\mathrm{P}(\mu,\tau)-1, \tag{27}$$

where $\mathrm{P}(\mu,\tau) := \frac{\mu^{\frac{\alpha-1}{\alpha}}(1+\beta)(1-\alpha+\tau)}{(1-\alpha)[1+\beta(1+\tau)]}$, $g'_\mu(\mu,\tau)<0$ and $g'_\tau(\mu,\tau)>0$, that is the rate of economic growth is a monotonic decreasing (increasing) function of the wage mark up (consumption tax).

[21] This extensions follow the suggestions of three anonymous referees.

Now, define

$$\tilde{\tau} := \frac{(1-\alpha)(1+\beta)\left(1-\mu^{\frac{1-\alpha}{\alpha}}\right)}{\mu^{\frac{1-\alpha}{\alpha}}\beta(1-\alpha)-(1+\beta)}, \tag{28}$$

as the threshold value of the consumption tax such that $\mathrm{P}(\mu,\tau)=1$, where $\hat{\tau} > \tilde{\tau}$, and

$$\tilde{\mu} := \left[\frac{1+\beta}{\beta(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}}, \tag{29}$$

as the threshold value of the wage mark up below which $\tilde{\tau} > 0$.

Although a rise in μ is now always growth-reducing, the following proposition shows that minimum wages used in conjunction with unemployment benefits can stimulate growth.

Proposition 3. *Let $1<\mu<\tilde{\mu}$ and $\tilde{\tau}<\tau<\hat{\tau}$ hold. Then $g(\mu,\tau)>g_c$ and the unemployment benefit policy is feasible.*

Proof. From Eq. (27) it is easy to verify that $g(\mu,\tau)=g_c$ if and only if $\mathrm{P}(\mu,\tau)=1$. Since $\mathrm{P}(\mu,\tau)>1$ for any $\tau>\tilde{\tau}$, $\tilde{\tau}>0$ for any $1<\mu<\tilde{\mu}$, and the unemployment benefit policy is feasible if and only if $\tau<\hat{\tau}$, then Proposition 2 follows. **Q.E.D.**

5.2. Skilled and unskilled labour

In this section we assume that two types of labour do exist: skilled (S) and unskilled (U). The market for skilled labour is competitive and the market for unskilled labour is regulated by law. We take the division of labour exogenously and assume that each young is endowed with θ units of unskilled time and $1-\theta$ units of skilled time (inelastically supplied on the markets for unskilled and skilled labour, respectively), where $0<\theta<1$ (see, e.g., Martínez and Iza, 2004). Therefore,

$N = N^U + N^S$, where $N^U = \theta N$ and $N^S = (1-\theta)N$. For the skilled time individuals earn a unitary competitive wage w_t^S. For the unskilled one they earn instead a minimum wage $w_{m,t}^U := \mu w_{c,t}^U$ fixed by law if employed, where $w_{c,t}^U$ is the competitive wage of the unskilled, while receiving a benefit $b_t := \gamma w_{c,t}^U$ if unemployed. Of course, $w_t^S > w_{m,t}^U$. Preferences over young-aged and old-aged consumptions are still determined by Eq. (1). Therefore, aggregate savings are now given by:

$$s_t = \frac{\beta}{1+\beta}\left\{(1-\theta)w_t^S + \theta\left[w_{m,t}^U(1-u_t) + b_t u_t\right]\right\}, \tag{30}$$

where $u_t = \left(\theta N - L_t^U\right)/\theta N$ is the unemployment rate in an economy with heterogeneous labour and L_t^U is the demand for raw labour.

There are three factors of production: physical capital (K), skilled (L^S) and unskilled (L^U) labour. The production function of the single firm i at time t now is $Y_{i,t} = K_{i,t}^{\alpha_1}\left(L_{i,t}^S\right)^{\alpha_2}\left(L_{i,t}^U\right)^{\alpha_3}\left(A_{i,t}\right)^{\alpha_2+\alpha_3} = \overline{\overline{B}} k_t^{1-\alpha_1} K_{i,t}^{\alpha_1}\left(L_{i,t}^S\right)^{\alpha_2}\left(L_{i,t}^U\right)^{\alpha_3}$, where $\alpha_1 + \alpha_2 + \alpha_3 = 1$, $\alpha_2 > \alpha_3$ and the labour productivity index $A_{i,t}$ is defined as in Section 2.2, with $\overline{\overline{B}} := a^{1-\alpha_1} > 0$. Aggregate production at t therefore takes place according to $Y_t = \overline{\overline{B}} k_t^{1-\alpha_1} K_t^{\alpha_1}\left(L_t^S\right)^{\alpha_2}\left(L_t^U\right)^{\alpha_3}$, where $L_t^S = N^S = (1-\theta)N$ and $L_t^U = (1-u_t)N^U = (1-u_t)\theta N$.[22] The intensive form production function is $y_t = \overline{\overline{B}} k_t (1-u_t)^{1-\alpha_1-\alpha_2}(1-\theta)^{\alpha_2}\theta^{1-\alpha_1-\alpha_2}$. Profit maximisation implies:

$$r_t = \alpha_1 \overline{\overline{B}}(1-u_t)^{1-\alpha_1-\alpha_2}(1-\theta)^{\alpha_2}\theta^{1-\alpha_1-\alpha_2} - 1, \tag{31}$$

[22] This production function implies limited substitutability between all production factors (see, e.g., Lindh and Malmberg, 1999; Fanti and Manfredi, 2005). The question of the degree of substitutability between production inputs is still controversial in literature. Although recent contributions provided evidence in favour of capital-skill complementarities (e.g., Hamermesh, 1993; Duffy et al., 2004), in this context such an assumption would lead to a lack of mathematical tractability. The important investigation of considering capital and skilled labour to be more complement then capital and unskilled labour in an economy with minimum wages is left for future research.

$$w_t^S = \alpha_2 \overline{\overline{B}} k_t (1-u_t)^{1-\alpha_1-\alpha_2} (1-\theta)^{\alpha_2 - 1} \theta^{1-\alpha_1-\alpha_2}, \tag{32}$$

$$w_{m,t}^U = (1-\alpha_1-\alpha_2) \overline{\overline{B}} k_t (1-u_t)^{-(\alpha_1+\alpha_2)} (1-\theta)^{\alpha_2} \theta^{-(\alpha_1+\alpha_2)}. \tag{33}$$

Since $w_{c,t}^U = (1-\alpha_1-\alpha_2)\overline{\overline{B}} k_t (1-\theta)^{\alpha_2} \theta^{-(\alpha_1+\alpha_2)}$, then from Eq. (33) the unemployment rate is

$$u_t = u(\mu) = 1 - \mu^{\frac{-1}{\alpha_1+\alpha_2}}. \tag{34}$$

Therefore, a rise in μ increases unemployment and reduces both the interest rate and skilled wage, and thus it also decreases the ratio of the skilled wage to the unskilled one.[23] Moreover, since a rise in γ does not affect unemployment it leaves the ratio $w_t^S / w_{m,t}^U$ unaltered. Therefore, the ratio of skilled to unskilled wage is greater when skilled labour is relatively scarcer (see, e.g., Acemoglu and Zilibotti, 2001), i.e. when θ is relatively high.

The government budget is $\theta b_t u_t = \tau_t (c_{1,t} + c_{2,t})$ and thus the budget-balancing consumption tax rate now becomes:

$$\tau_t = \tau(\mu) = \frac{\gamma(1-\alpha_1-\alpha_2)(1+\beta)\left(\mu^{\frac{1}{\alpha_1+\alpha_2}} - 1\right)}{\mu(1+\alpha_1\beta) - \gamma(1-\alpha_1-\alpha_2)\beta\left(\mu^{\frac{1}{\alpha_1+\alpha_2}} - 1\right)}. \tag{35}$$

which is similar to Eq. (12) and where $\tau(1) = 0$, $\tau'(\mu) > 0$ and $\tau > 0$ for any $1 < \mu < \mu_{\mathrm{MM}}$ (with μ_{MM} being the new threshold value of the wage mark up below which the unemployment benefit policy is feasible in the case of skilled and unskilled labour).

Since equilibrium in goods and capital markets is still determined by Eq. (13), the growth rate of the economy can now be expressed as:

$$\overline{\overline{g}}(\mu) = (1+\overline{\overline{g}}_c)\overline{\overline{\mathrm{H}}}(\mu) - 1, \tag{36}$$

[23] Notice that this result is similar to that obtained by Sener (2006, Proposition 1, p. 785), which, in an open economy model à la Ramsey, focuses on the effects of minimum wages on (*i*) unemployment, (*ii*) the relative wage skilled-unskilled, and (*iii*) the R&D intensity, but neglecting however both growth and welfare issues.

where $\overline{\overline{g}}_c = \frac{\beta}{1+\beta}\overline{\overline{B}}(1-\alpha_1)(1-\theta)^{\alpha_2}\theta^{1-\alpha_1-\alpha_2} - 1$ is the growth rate under *laissez-faire* in an economy with labour heterogeneity, $\overline{\overline{\mathrm{H}}}(\mu) := \mu^{\frac{\alpha_1+\alpha_2-1}{\alpha_1+\alpha_2}} + \gamma\frac{1-\alpha_1-\alpha_2}{1-\alpha 1}\left(1-\mu^{\frac{-1}{\alpha_1+\alpha_2}}\right)$ and $\overline{\overline{g}}(1) = \overline{\overline{g}}_c$.

Let

$$\overline{\overline{\gamma}} := 1-\alpha_1, \tag{37}$$

be a threshold value of the replacement rate. Then we have the following proposition.

Proposition 4. *Let* $1 < \mu < \mu_{\mathrm{MM}}$ *hold to guarantee feasibility of the unemployment benefit policy. (1) If* $\gamma \le \overline{\overline{\gamma}}$*, then* $\overline{\overline{g}}(\mu) < \overline{\overline{g}}_c$*. (2) If* $\gamma > \overline{\overline{\gamma}}$*, then* $\overline{\overline{g}}(\mu) > \overline{\overline{g}}_c$ *for any* $1 < \mu < \mu^{\circ\circ}$*,* $\overline{\overline{g}}(\mu)$ *is maximised at* $\mu = \hat{\hat{\mu}}$ *and* $\overline{\overline{g}}(\mu) < \overline{\overline{g}}_c$ *if* $\mu > \mu^{\circ\circ}$*, where*

$$\hat{\hat{\mu}} := \frac{\gamma}{\overline{\overline{\gamma}}} > 1, \tag{38}$$

and $\mu^{\circ\circ} > \hat{\hat{\mu}}$ *is the value of the wage mark up such that* $\overline{\overline{g}}(\mu^{\circ\circ}) = \overline{\overline{g}}_c$*.*

Proof. Differentiating Eq. (36) with respect to μ gives $\overline{\overline{g}}'(\mu) = (1+\overline{\overline{g}}_c)\overline{\overline{\mathrm{H}}}'(\mu)$, where $\overline{\overline{\mathrm{H}}}'(\mu) = \frac{1-\alpha_1-\alpha_2}{(1-\alpha_1)(\alpha_1+\alpha_2)}\mu^{\frac{-(1+\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}}(\gamma - \mu\overline{\overline{\gamma}})$ and thus $\mathrm{sgn}\{\overline{\overline{g}}'(\mu)\} = \mathrm{sgn}\{\overline{\overline{\mathrm{H}}}'(\mu)\}$. Then $\overline{\overline{\mathrm{H}}}'(\mu) \gtrless 0$ if $\mu \lessgtr \hat{\hat{\mu}}$. Therefore, Proposition 3 follows, since (1) if $\gamma \le \overline{\overline{\gamma}}$, $\overline{\overline{\mathrm{H}}}'(\mu) < 0$ for any $1 < \mu < \mu_{\mathrm{MM}}$, and (2) if $\gamma > \overline{\overline{\gamma}}$, the fact that $\overline{\overline{\mathrm{H}}}'(\mu) = 0$ only at $\mu = \hat{\hat{\mu}}$ and $\overline{\overline{\mathrm{H}}}''(\mu) = 0$ only at $\mu = \hat{\hat{\mu}}(1+\alpha_1+\alpha_2) > \hat{\hat{\mu}}$ complete the proof because $\overline{\overline{\mathrm{H}}}(\mu) = 1$ twice at $\mu = 1$ and $\mu = \mu^{\circ\circ}$. **Q.E.D.**

Proposition 4 shows that in an economy with heterogeneous labour the same parametric conditions of an economy with homogeneous labour are involved in determining whether a minimum wage can foster growth. In fact, comparison of Eqs. (19) and (37) reveals that the replacement rate beyond which the minimum wage is beneficial to growth still depends on the size of the output elasticity of capital. Moreover, Eq. (38) shows that the growth-maximising wage mark-up is now determined as the ratio between the replacement rate and the sum of the weight of skilled and unskilled labour in production, rather than the weight of (homogeneous) labour. This means that labour heterogeneity does not alter any of the conclusions of the baseline model.

The model built on in Section 2 as well as that of Section 5.2 represent of course a theoretical abstraction on the role of an important labour market institution such as the legally set minimum wage in affecting growth and welfare in a stylised economy, as wage minima have been assumed to be determined as a mark up upon (*i*) the prevailing equilibrium competitive wage, in the case of homogeneous labour (Section 2), and (*ii*) the prevailing equilibrium competitive wage in the market for raw labour, in the case of labour heterogeneity (Section 5.2). In real economies, however, the minimum wage is computed as a fraction of average earnings, and beneficiaries are essentially the poorest among low-paid workers. One of the most widely accepted measure of the impact of wage floor is the weighted Kaitz index. On the basis of such an index, minimum wages in Europe are found to be higher than in the U.S. (see Dolado et al., 1996, Table 1, p. 322–323), ranging between one third and one half[24] (see also OECD, 2008 and ETUI Policy Brief, 2009 for more recent evidence).

At the moment of writing the present paper, there exists a wide consensus at the European level to raise as well as to make the minimum wage uniform for citizens of several European Union countries in a measure even higher than the 50 per cent of average earnings (see, e.g., Schulten and

[24] The Kaitz index for Italy is even much higher. However, a national legislated (minimum) wage floor in this country does not exist, while being determined as a bargaining between unions and employers in sectoral collective agreements, and then it may greatly differ from sector to sector because of the existence of differentiated minimum wage rates.

Watt, 2007).[25] The question of whether introducing a coordinated European minimum wage policy is debated essentially to reinforce labour market institutions and protect workers against the severe recession recently experienced. Moreover, even in some States of the U.S. the minimum wage has been recently set at a level higher than the federal one as a measure to combat recession. Therefore, the question now is the following: would the positive effects of minimum wages on growth be confirmed in a simple overlapping generations endogenous growth model à la Romer with heterogeneous labour, in the case wage minima were realistically computed as a fraction of average wages?

For a simple quantitative illustration of the theoretical predictions of the model we now refer to an example of a European country where opponents and proponents of the existing minimum wage indexation mechanisms battled in the recent years, i.e. the case of France.[26] In order to answer the question above through a calibrated numerical exercise we assume first the (minimum) wage floor for low-paid workers to be computed as a percentage of the weighted average wage between skilled and unskilled labour when $u = 0$, that is $w_{m,t}{}^{U} := z \cdot \left[(1-\theta)w_{c,t}{}^{S} + \theta w_{c,t}{}^{U}\right]$, where $0 < z < 1$. Therefore, the wage gap in an economy with full employment is $\frac{w_{c,t}{}^{S}}{w_{c,t}{}^{U}} = \frac{\alpha_2}{1-\alpha_1-\alpha_2}\frac{\theta}{1-\theta}$. Second, from EUROSTAT (2005) Data (e.g., Table A.2, Hipolito, 2008), we take the 90-10, 50-10 and 90-50 differentials (i.e., the ratios of the 10th, 50th and the 90th deciles) of the wage distribution are 3.36, 1.64 and 2.00, respectively.[27] Third, by assuming for simplicity a uniform distribution within the

[25] In particular, "… the core proposal [of a European-wide minimum wage floor should involve] … an undertaking by all countries to raise, within a given time scale, their minimum wage to initially 50% and subsequently 60% of the average wage." (ETUI Policy Brief, 2009, p. 5).

[26] See Cahuc et al. (2008) and Askenazy (2008) for debates about the policy effects of the minimum wage amongst French economists.

[27] Note that important papers dealing with wage inequality for United States and other developed countries are Blau and Kahn (1996), Goldin and Katz (2007), Autor et al. (2008).

50-10 and 90-50 deciles, we obtain an approximated value of the ratio between the lowest wage (i.e., the wage of the 10 per cent of the low-paid) and average wage around 2.00. Since France has a current statutory minimum wage of about the 45-50 per cent of the average wage (see OECD, 2008; ETUI Policy Brief, 2009),[28] then this would approximately correspond to a wage floor covering the 10 per cent of the low-paid.

Therefore, taking seriously into account recent proposals to attempt to raise the minimum wage in several European countries up to 60 per cent of the average wage (see Footnote 25), we easily see that this would push the wage floor[29] up of about the 20 per cent, so that the preceding measure of wage inequality 50-10 and 90-10 deciles are in the short run compressed from 1.64 to 1.36 and from 3.36 to 2.8, respectively. What the consequences in terms of growth of raising the statutory minimum wage from 45-50 to 60 per cent would be in our context?

Our exercise is the following. Since the minimum wage covers the 10 per cent of workers among the low-paid we assume $\theta = 0.1$. Then we take the output elasticity of capital for France from Rodríguez and Ortega (2006, Table A.1), so that $\alpha_1 = 0.5$, and calibrate $\alpha_2 = 0.473$ such that the wage gap (i.e., the ratio between the average and lowest wages) is $\frac{w_{c,t}^{S}}{w_{c,t}^{U}} = 2$. It is therefore implicitly assumed that the current wage of the unskilled is the competitive one, that is the existing (minimum) wage floor (equal to the 50 per cent of the average wage) is not binding. The production

[28] More precisely, ETUI Policy Brief (2009, Table 1) reports the 51.4 per cent from OECD data and the 48.3 per cent from ILO data for the year 2007.

[29] Note that it is assumed – for illustrative purposes – that the wage floor only regards the first deciles of the whole population. Therefore in our model with only two categories, the share of unskilled (earning the lowest wage) is 0.1, while the remaining 0.9 is assumed to be skilled (thus earning the average wage above calculated, which is approximately double the amount of the wage earned by the lowest 10 per cent of the low-paid). Note that current statutory minimum wages in Europe only cover a very low share of workers, ranging between 1 and 5 per cent in several countries, so that the conjecture that a rise in the statutory value up to 60 per cent of the average wage to cover the 10 per cent of workers seems to be rather realistic.

scale parameter ($B = 60$) is fixed to get a reasonable growth rate around 1.7 per cent per annum in the case of full employment. The subjective discount ($\beta = 0.11$), instead, is calibrated to obtain a propensity to save around the 10 per cent (see, e.g., Jappelli and Padula, 2007, Table 1 for the case of France). The replacement rate γ is assumed to be 0.6.[30]

A simple calculation therefore shows that if the statutory minimum wage were raised as proposed from 50 to 60 per cent of the average wage, the (minimum) wage of the unskilled will raise from $w_{c,t}^{U} = 14.13$ to $w_{m,t}^{U} = 16.2$ (i.e. almost the 20 per cent above the wage of the unskilled in the competitive-wage economy; this corresponds to $\mu = 1.14$ in our context) and the per annum growth rate (assuming each generation consists of 30 years) will increase from 1.705 per cent in the competitive wage economy to 1.712 per cent in the regulated-wage economy. The unemployment rate is reasonably at the 12.5 per cent and the consumption tax rate to finance the unemployment benefits is negligible at the 0.24 per cent. Moreover, the rise in the growth rate also raises the skilled wage because of the higher capital accumulation; as a consequence, and thus not only the current unskilled but also all the future skilled will beneficiate of the higher minimum wage.

The numerical examples above, therefore, revealed that the minimum wage, even when realistically computed as a fraction of average earnings, and used together with a system of unemployment benefits, should be used as an instrument to promote growth and welfare. Of course, since our model economy is highly stylised, policy conclusions that can be derived should be carefully evaluated. However, since an indirect positive relationship between unemployment and growth certainly exists from a theoretical viewpoint, our results shed light on the beneficial effects the minimum wage can play in a simple and intuitive framework such as the one-sector Cobb-Douglas endogenous growth model à la Romer.

[30] The amount of the ARE (return to work credit) in France varies according to the wage received by the jobseeker during the reference period. The gross amount of the unemployment benefit is equal to the highest of the following two sums: 57.4 per cent of the SJR (reference daily salary) or 40.4 per cent of the SJR plus 11.04 euro per day. Then, in our context a replacement rate around 55-60 per cent can be realistic.

5.3. Capital income tax

As is known, in an OLG context taxing capital income may stimulate growth because it causes a positive inter-generational transfer effect towards the young savers (see Bertola, 1996; Uhlig and Yanagawa, 1996). In this section we briefly show that financing unemployment benefits with capital income taxes, rather than consumption taxes, produces beneficial effects on growth even if the range of values of wage mark up such that the regulated-wage economy grows faster than the competitive-wage economy is reduced with respect to the baseline model analysed in the previous sections.

Since the propensity to save in independent of the interest rate with Cobb-Douglas preferences (see Eq. 6.2), the growth rate of the economy is still given by Eq. (18). The government budget, instead, becomes:

$$b_t u_t = \tau_{k,t} r_t k_t , \tag{39}$$

where $0 < \tau_{k,t} < 1$ is the capital income tax rate. Using Eqs. (2.2), (9.1), (10.1) and the one-period backward Eq. (13), the (constant) budget-balancing capital income tax rate is:

$$\tau_{k,t} = \tau_k(\mu) = \frac{\gamma\left(\mu^{\frac{1}{\alpha}} - 1\right)(1-\alpha)B}{\alpha B\mu - 1} . \tag{40}$$

From Eq. (40), the unemployment benefit policy is feasible if and only if for any $0 < \gamma < 1$, $1 < \mu < \mu_R$ holds, where μ_R (that cannot be solved in closed form) is the wage mark up such that $\tau_k(\mu) < 1$. Therefore, we have the following proposition.

Proposition 5. *Let* $1 < \mu < \mu_R$ *hold to guarantee feasibility of the unemployment benefit policy. Then, conclusions of Proposition 1 still remain valid.*

Proof. Since Eq. (18) holds and $1<\mu<\mu_R$ guarantees $\tau_k(\mu)<1$ then Proposition 5 follows. **Q.E.D.**

To illustrate Proposition 5 we take the same parameter values as in Section 3. The growth-maximising wage mark up, therefore, still remains $\hat{\mu}=1.272$, so that $g(\hat{\mu})=1.627$ and $u(\hat{\mu})=0.414$. The budget-balancing capital income tax is $\tau_k(\hat{\mu})=0.522$, which is higher than in the case of consumption taxes ($\tau(\hat{\mu})=0.26$). Moreover, the range of wage up for which the unemployment benefit policy is feasible shrinks from $1<\mu<\mu_{\mathrm{M}}=6.568$ in the case of consumption taxation to $1<\mu<\mu_R=1.562$ in the case of capital income taxation. The beneficial effects on growth of using the minimum wage combined with unemployment benefits financed with capital income taxes rather than consumption taxes are therefore reduced.

The analyses presented in this section, therefore, revealed that the beneficial effects of the minimum wage on growth (with tax-financed unemployment benefits) are a robust feature of OLG economies with both homogeneous and heterogeneous labour.

6. Conclusions

This paper takes a dynamic view on labour market rigidities. Analysis of labour market imperfections and the effects of unemployment in aggregate macroeconomic models have been widely studied in the economic literature. As regards how legislated wage minima affect economic growth, conclusions are essentially negative unless when the minimum wage causes positive external effects (for instance, on the accumulation of human capital, the R&D sector and so on) or when capital and labour are relatively complement in production, i.e. the elasticity of substitution between factor inputs is relatively low.

In contrast to the previous theoretical literature, in this paper we introduce minimum wage and unemployment benefit policies in a simple double Cobb-Douglas overlapping generations growth model à la Romer (1986). It is shown that a regulated-wage economy with unemployment may grow more rapidly than the *laissez-faire*, and a growth-maximising minimum wage exists in the case of both homogeneous and heterogeneous labour. We have also shown that the growth-promoting effect of the minimum wage is robust to different unemployment benefit policies (such as financing the benefit system either with consumption taxes or capital income taxes as well as by assuming the replacement rate or the tax rate as an endogenous variable to balance out the government budget). Moreover, since the minimum wage in real economies, especially in Europe, is computed as fraction of average earnings, in the model with heterogeneous labour we presented a calibrated numerical example for the case of France, where the debate between economists and politicians about both the macroeconomic effects and indexation mechanisms of wage floor were heated recently, to show the beneficial effects on growth of raising the minimum wage up to the sixty per cent of the average wage.

Moreover, we found that minimum wages can be welfare-improving and the highest possible long-run welfare level is achieved when balanced growth is maximised. In particular, we identified the conditions under which the minimum wage can make the current as well as all the future (infinite) generations better off.[31]

The essential message of the present paper, therefore, is that minimum wage policies can be used not only for equity reasons but also to promote economic growth and welfare even in the absence of positive external effects the minimum wage may create.

Our findings contribute to the economic literature framed in the basic one-sector endogenous growth model and have straightforward policy implications. In particular, the present paper may

[31] Interestingly, we have also analysed how a progressive rise in the minimum wage (see the cases of Ireland and U.K.) affect growth and welfare in contrast to the once-and-for-all policy. With a gradual rise in the wage floor we found that it is more likely that the generations from the current one onwards are better off (although the minimum wage it cannot be Pareto optimal because the current old-aged suffer due to the unemployment occurrence and the corresponding reduction of the rate of interest).

complement the paper by Irmen and Wigger (2002), where, in an overlapping generations context similar than ours, but considering a right-to-manage union that fixes wages, rather than a legislated minimum wage, and without assuming unemployment benefit policies, a positive relationship between unionised wages, unemployment and economic growth is established. Their result, however, holds only when capital and labour are enough complement in production, and thus it is prevented in the of case Cobb-Douglas technology, while our findings are confirmed (*a fortiori*) in the case of complementarity between production factors.

The present paper can be extended in several directions. For instance, utility and production functions can be generalised and an open economy framework used to assess the role the minimum wage can play on economic growth and welfare in such a case. Moreover, fertility can be endogenised with individuals being free to choose both the quality and quantity of children and then accumulate human capital through education. Finally, deficit and debt policies[32] could be incorporated to enrich the model and test for robustness of the minimum wage policy in a dynamic context.

References

Acemoglu D, Zilibotti F (2001) Productivity differences. Q J Econ 116:563–606.

Aghion P, Howitt P (1994) Growth and unemployment. Rev Econ Stud 61:477–494

Askenazy P (2003) Minimum wage, exports and growth. Eur Econ Rev 47:147–164

Askenazy P (2008) Est-il urgent de réformer le SMIC? Alternatives Economiques 269:56–57

[32] For instance, Josten (2006) analysed deficit and debt policies in a model with unionised wages.

Autor DH, Katz LF, Kearney MS (2008) Trends in U.S. wage inequality: revising the revisionists. Rev Econ Statist 90:300–323

Bertola G (1996) Factor shares in OLG models of growth. Eur Econ Rev 40:1541–1560

Blanchard OJ (1998) Revisiting European unemployment: unemployment, capital accumulation and factor prices. NBER Working Paper No 6566

Blau FD, Kahn LM (1996) International differences in male wage inequality: institutions versus market forces. J Polit Econ 104:791–837

Cahuc P, Michel P (1996) Minimum wage unemployment and growth. Eur Econ Rev 40:1463–1482

Cahuc P, Cette G, Zylberberg A (2008) Salaire minimum et bas revenus: comment concilier justice sociale et efficacité économique? Les Rapports du Conseil d'analyse économique No 79 La Documentation française, Paris

Card D, Krueger AB (1994) Minimum wages and employment: a case study of the fast-food industry in New Jersey and Pennsylvania. Am Econ Rev 84:772–793

Card D, Krueger AB (1995) Myth and Measurement: The New Economics of the Minimum Wage. Princeton University Press, Princeton NJ

Card D, Krueger AB (1998) A reanalysis of the effects of the New Jersey minimum wage increase on the fast-food industry with representative payroll data. NBER Working Paper No 293

Corneo G, Marquardt M (2000) Public pensions, unemployment insurance and growth. J Public Econ 75:293–311

Daveri F, Tabellini G (2000) Unemployment, growth and taxation in industrial countries. Econ Pol 15:49–104

de la Croix D, Michel P (2002) A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations. Cambridge University Press, Cambridge

Diamond PA (1965) National debt in a neoclassical growth model. Am Econ Rev 55:1126–1150

DiNardo JE, Fortin NM, Lemieux T (1996) Labor market institutions and the distribution of wages, 1973-1992: a semiparametric approach. Econometrica 64:1001–1044

Dolado J, Kramarz F, Machin S, Manning A, Margolis D, Teulings C (1996) The economic impact of minimum wages in Europe. Econ Pol 11:317–372

Duffy J, Papageorgiou C, Perez-Sebastian F (2004) Capital-skill complementarity? Evidence from a panel of countries. Rev Econ Statist 86:327–344

Falk A, Fehr E, Zehnder C (2006) Fairness perceptions and reservation wages – The behavioral effects of minimum wage laws. Q J Econ 121:1347–1381

Fanti L, Manfredi P (2005) Age heterogeneities, growth and distribution in a Solow-Stiglitz model. In: Salvadori N, Panico C (eds.), Distribution and Growth, Edward Elgar, Cheltenham

Fortin NM, Lemieux T (1997). Institutional change and rising wage inequality. J Econ Perspect 11:75–96

Goldin C, Katz LF (2007) Long-run changes in the wage structure: narrowing, widening, polarizing. Brookings Pap Econ Act 2:135–165

Hamermesh DS (1993) Labor Demand. Princeton University Press, Princeton NJ

Hellwig MF, Irmen A (2001) Wage growth, productivity growth, and the evolution of employment. CEPR Discussion Paper No 2927

Hipolito S (2008) International differences in wage inequality: a new glance with European matched employer-employee data. MPRA Working Paper No 7932

Irmen A, Wigger BU (2002) Trade union objectives and economic growth. FinanzArch 59:49–67

Irmen A, Wigger BU (2006) National minimum wages, capital mobility, and global economic growth. Econ Lett 90:285–289

Jappelli T, Padula M (2007) Households' saving and debt in Italy. CSEF Working Paper No 183

Josten SD (2006) Dynamic fiscal policies and unemployment in a simple endogenous growth model. Int Tax Public Finance 13:701–716

Katz LW, Murphy KM (1992) Changes in relative wages, 1963–1987: supply and demand factors. Q J Econ 107:35–78

Kraay A, Raddatz C (2007) Poverty traps, aid, and growth. J Dev Econ 82:315–347

Lindh T, Malmberg B (1999) Age structure effects and growth in the OECD, 1950-1990. J Population Econ 12:431–449

Lucas RJ (1988) On the mechanics of economic development. J Monet Econ 22:2–42

Martínez DF, Iza A (2004) Skill premium effects on fertility and female labour force supply. J Population Econ 17:1–16

Meckl J (2004) Accumulation of technological knowledge, wage differentials, and unemployment. J Macroecon 26:65–82

Müller KU, Steiner V (2008) Would a legal minimum wage reduce poverty? A microsimulation study for Germany. IZA Discussion Paper No 3491

Neumark D, Wascher W (1995) Minimum-wage effects on school and work transitions of teenagers. Am Econ Rev 85:244–249

Neumark D, Wascher W (2006) Minimum wages and employment: a review of evidence from the new minimum wage research. NBER Working Paper No 12663

OECD (1998) Employment Outlook, Paris

OECD (2008) Employment Outlook, Paris

Ono T (2007) Unemployment dynamics in an OLG economy with public pensions. Econ Theory 33:549–577

Ravn MO, Sørensen JR (1999) Schooling, training, growth and minimum wages. Scand J Econ 101:441–457

Rodríguez F, Ortega D (2006) Are capital shares higher in poor countries? Evidence from Industrial Surveys. Wesleyan Economics Working Papers No 2006-023

Romer PM (1986) Increasing returns and long-run growth. J Polit Econ 94:1002–1037

Schulten T, Watt A (2007) European minimum wage policy – A concrete project for a social Europe. European Economic and Employment Policy Brief, Brussels. ETUI: http://www.etui

Sener F (2006) Labor market rigidities and R&D-based growth in the global economy. J Econ Dynam Control 30:769–805

Shimer R, Werning I (2006) Reservation wages and unemployment insurance. Q J Econ 122:1145–1185

Stigler GJ (1946) The economics of minimum wage legislation. Am Econ Rev 36:358–365

Tamai T (2009) Inequality, unemployment, and endogenous growth in a political economy with a minimum wage. J Econ 97:217–232

Uhlig H, Yanagawa N (1996) Increasing the capital income tax may lead to faster growth. Eur Econ Rev 40:1521–1540

Vedder RK, Gallaway LE (2001) Does the minimum wage reduce poverty? Employment Policy Institute Working Paper No 06-2001